Test Yourself – Mathematics (Solutions)

1)

a) Y = C0 + C1·Y + I + G
   Y – C1·Y = C0 + I + G
   Y(1 – C1) = …
   Y = (C0 + I + G)/(1 – C1)

b) A = $[X \cdot B + (1 - X) \cdot C]^{1/2}$
   $A^2 = X \cdot B + (1 - X) \cdot C = X \cdot B + C - X \cdot C = X(B - C) + C$ *(≥ 0)*
   $X = (A^2 - C)/(B - C)$ *(B – C ≠ 0)*

2) (One way of solving the problem)

$$\left\{\begin{array}{l|l} X_1 + X_2 = 1 & \\ X_1 \cdot 6 - X_2 \cdot 3 = 6 & \cdot (1/3) \end{array}\right.$$

$$X_1 \cdot (1+2) = 1 + 2 \Rightarrow X_1 = 1$$

$$X_2 = 1 - X_1 = 1 - 1 = 0$$

3)

a) R = –4% r = ln(1-0.04) = ln 0.96 = –4.08%
b) r = 5% R = e^r –1 = 5.13%

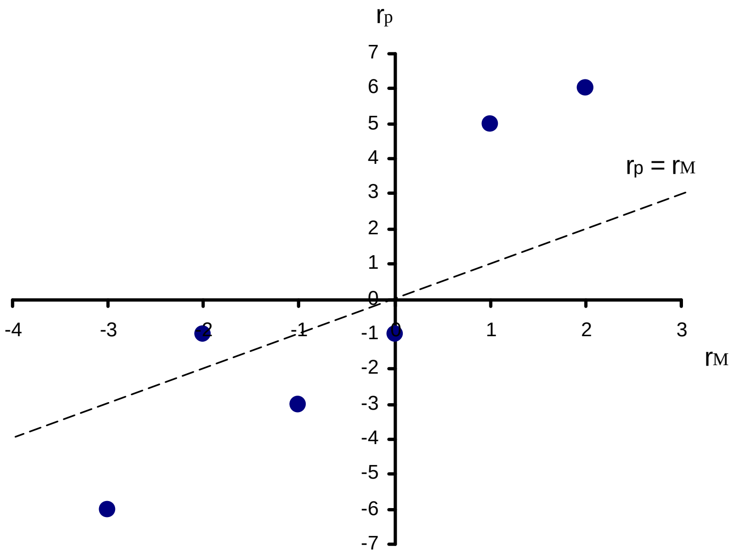
4) $k \cdot P = P \cdot (1+0.10)^n$ (n = number of years) $\Rightarrow$ $k = (1.10)^n$
Using the logarithm:
$\ln(k) = \ln[(1.10)^n] = n \cdot \ln(1.10)$ $\Rightarrow$ n = ln(k)/ln(1.10)
with k = 2 : n = ln(2)/ln(1.10) = 7.27 years

5) a) The following table summarises the solution:

t	R	r	P_t
0	-	-	200
1	-5.000	-5.129	190
2	10.526	10.001	210
3	9.524	9.097	230
4	–8.696	–9.097	210
5	4.762	4.652	220
$\sum_{t=1}^{5}$	11.116	9.524	

b) $\bar{R} = \dfrac{11.116}{5} \approx 2.22\%$, $\bar{r} = \dfrac{9.524}{5} \approx 1.90\%$ (or 1.91% using the rounded values in he table above). The mean of the simple returns overestimates the effective mean return which is about 1.92% (= exp(0.0190)-1)! The proper mean is the arithmetic mean of the continuous returns re-transformed into simple returns.

6) a) Graphique:



If the market (=benchmark) yields a positive return, then the portfolio yields a return strictly above the benchmark return (the dashed line indicates equality of the two returns); if the market (=benchmark) yields a negative return, then the portfolio return as mostly below the benchmark return. It thus seems that the portfolio conserves a relatively constant beta that is above 1, i.e., the points are roughly scattered along a line with a slope greater than 1 (so that it is steeper than the dashed line).

b) c) and d)

Calculation of the means and standard deviations, of the covariance and the correlation coefficient:

t	1	2	3	4	5	6	□
$r_{P,t}$	-3	5	-1	-6	6	-1	**0**
$(r_{P,t} - \bar{r}_p)^2$	9	25	1	36	36	1	**108**
$r_{M,t}$	-1	1	0	-3	2	-2	**-3**
$(r_{M,t} - \bar{r}_M)^2$	0.25	2.25	0.25	6.25	6.25	2.25	**17.5**
$(r_{P,t} - \bar{r}_p)(r_{M,t} - \bar{r}_M)$	1.5	7.5	-0.5	15	15	1.5	**40**

Means: □ $r_p = 0$ □ $\bar{r}_p = 0/6 = 0\%$

□ $r_M = -3$ □ $\bar{r}_M = -3/6 = -0.5\%$

Standard deviations: $\sigma_p = [\frac{1}{6-1} 108]^{½} = 21.6^{½} \approx 4.65\%$

$\sigma_M = [\frac{1}{6-1} 17.5]^{½} = 3.5^{½} \approx 1.87\%$

Covariance: $cov(r_{P,t}, r_{M,t}) = (1/5) \cdot 40 = 8.$

Correlation coefficient:
$r_{p,M} \approx 8/(4.65 \cdot 1.87) \approx 0.92$. Very strong correlation between the two! A linear regression is suitable to express the connection between the two.

e) $\beta = 8/3.5 \approx 2.286$; $\alpha \approx 0 - 2.286 \cdot (-0.5) = 1.143$.

f) $R^2 = r_{p,M}^2 \approx 0.92^2 \approx 0.85$ or equivalently : $R^2 = \beta^2 \cdot \sigma_M^2 / \sigma_p^2 \approx 5.23 \cdot 3.5/21.6 \approx 0.85$.
The regression coefficient indicates that using the regression about 85% of the variance of the investor's returns can be explained by the varince of the benchmark returns. The regression is thus suitable (significant) by the measure of R^2.

g) The regression yields the following equation: $r_{p,t} = 1.143 + 2.286 \cdot r_{M,t}$.
Thus if the forecasted return of the benchmark is $r_{M,t} = 2\%$, then the expected return for the investor is $r_{p,t} = 1.143 + 2.286 \cdot 2 = 5.715$ (%).

